Mathematics Tutor Workshop

Resources and Materials

T. Inoue, A. Setniker, J. Simons
Department of Sciences & Mathematics 2020

[redacted]

Summary

This material has been created and compiled for Mathematics Tutor training workshops to supplement tutor training at the CSU Maritime Academy. The resources herein are designed to be used in a workshop setting, with activities led by mathematics faculty.

Contents

CHAPTER 1

What are Teaching, Learning, and Pedagogy?

Before we get into tutoring speci cally, we need to cover some basic vocabulary and concepts relevant to the learning process. This is important to ensure we all can start with the same foundation.

- Teaching is the practice or art of imparting information from an instructor to a student. We like to think of this as an act of empowerment that expands intellectual and physical capabilities for students. Teaching refers to the instructor's role in the learning process. Teaching is not passive on the part of the instructor, and must be adapted to students and educational settings (classrooms, number of students/tables/desks/whiteboards/etc.)
- Learning is what happens on the student side: it is the expansion of a student's knowledge or skill base. It is not a passive process on the part of the student. Students must engage with the material in order to learn.
- Pedagogy refers to the practices of teaching. While the classic university pedagogical structure has historically been a lecture-style teaching method, many instructors adopt strategies that involve active learning within their classroom. Active learning refers to activities in which students must engage and participate in the process of discovery. This can be done with worksheets, board work, group work, and a number of other tools.
- Assessment refers to the measurement of student comprehension of a topic or learning objective using assignments or tasks.
- Learning objectives/outcomes refer to the content students are expected to learn in a course.

As a tutor in this workshop, our goals for you are:

1. To provide some contextual framework for how teaching and learning happen (and don't!);
2. To assess your strengths and weaknesses as an instructor;
3. To provide you with some pedagogical tools to improve your e ectiveness as a tutor;
4. To provide you with an inventory for our mathematics courses that can help you assess your own knowledge base and nd information; and

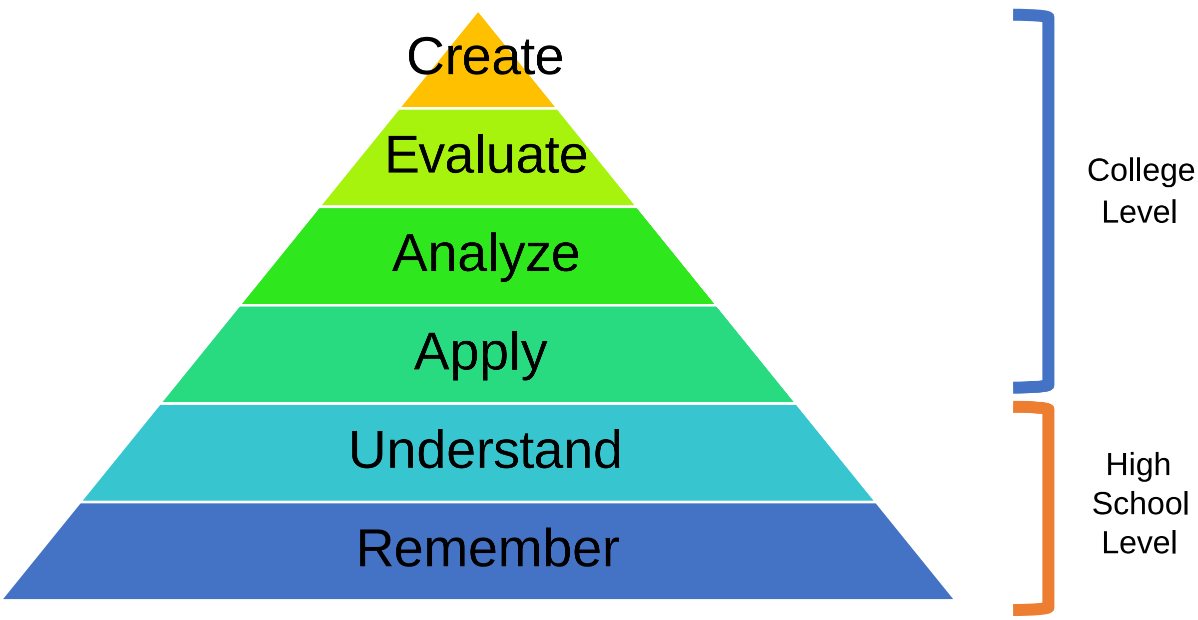
Learning

On the next pages, you'll find a summary of what is known as Bloom's Taxonomy, which is a classification system for how educators think of measuring student understanding of a concept.[1] One important aspect to note is that when teachers create assignments and tests, they often have this taxonomy in mind and are attempting to evaluate student performance on these specific levels. Instructors often aim to assess ALL levels of the taxonomy during a course as well as all student learning outcomes for the particular course. Notice the labeling of these learning levels in Bloom's Taxonomy: some are consider high school level and others college level. At universities, classes are often not geared towards simple memory exercises and repeating of ideas but instead towards more detailed analyses and expansion or creation of new ideas.

1

Levels of Understanding

Bloom's Taxonomy is a hierarchy of learning levels. It is best represented as a pyramid where the foundation of learning is shown at the bottom, with increasingly higher-order learning as you climb toward the top. The purpose of Bloom's Taxonomy is to make a distinction between different levels of learning so that students can study at appropriate learning levels. In high school, you might have been responsible for acquiring basic knowledge, while college courses require more application, analysis, and evaluation, and possibly even the creation of something new with what you are learning.



Most students come into college with strong study skills in how to remember and understand concepts. The challenge is to create new active study strategies that enable students to take what they know and process it at a higher level. It is important to study at the level at which you will be expected to demonstrate competency.

Recognizing Learning Levels

Directions: Using your knowledge of Bloom's Taxonomy, identify which level of understanding is required to answer the following statements/questions. (Refer to the Bloom's handout for clarification.)

- Remembering
- Understanding
- Applying
- Analyzing
- Evaluating
- Creating

1. Describe the significance of the Corps of Cadets to Cal Maritime.
2. What year was Cal Maritime founded?
3. Explain the role of the Commandants.
4. Suppose Cal Maritime wants to add a new college. What college would you create and why?
5. Your friend argues that Cal Maritime should not have joined the CSU. Do you agree? Why or why not?
6. Who was the first president of Cal Maritime?
7. Cal Maritime is a university defined by six core values. Break down each, describing how they relate to one another and create a unified University.
8. You meet Alicia, a classmate, and notice she is wearing her salt & peppers. What can you now conclude about Erin?
9. Why is a horizontal line a function while a vertical line is not?
10. How do we physically interpret the derivative?

CHAPTER 2

Tutoring Practices

So you've been hired as a tutor and we've talked a little bit about teaching vs. learning and learning taxonomy ... now what? You probably already have an idea of what tutoring looks like and how you might go about tutoring. You may have also received some general tutor training that set some of the ground rules and expectations for your position. The next phase of this is to prepare youslyoi9W28 10.2om

AN EASILY IMAGINED ENCOUNTER BETWEEN A TUTOR AND STUDENT

by

Taiyo Inoue

Taiyo Inoue
California Maritime
Academy

FADE IN:

INT. TUTORING CENTER

Alice and Bob meet at the tutoring center. Alice is staring at her phone while Bob is just arriving. The narrator is not physically present at the tutoring center -- think of her/him as the voice of God.

BOB
Hello there Alice!

ALICE
Oh, hi Bob.

BOB
I could sure use some help with MTH 100. My professor assigned some homework and I don't understand one of the problems.

ALICE
No sweat. What's the statement of the problem you are having trouble with?

BOB
Well, I'm given a table of values. Let me write it on the board.

Bob writes on the whiteboard the following table of values:

x	y
1	3
2	3
3	4
3	-4
4	3

Alice, feeling bored during the 15 second duration in which Bob is writing the table of values, pulls out her phone and giggles. Bob looks over at Alice and feels a little bit dejected.

BOB
The question is asking me if this table of values represents a func...tion.. or... uh,
Alice, are you listening?

NARRATOR

An effective tutor should demonstrate dedication and empathy. The phone feels like a rejection. Alice should try that again:

Rewind time.

Alice sits with her boredom while Bob writes the values on the board. Alice feels frustration at how slowly Bob is writing, but tries to practice patience.

BOB

The question is asking me if this table of values represents a function or not.

ALICE

Oh, that's easy. It's not a function. Done.

Alice returns to her phone and Bob happily writes down the answer, but still has no idea why he's writing what he's writing.

NARRATOR

This encounter was a failure. An effective tutor is never a mere dispensary for homework solutions. Rather, an effective tutor gently guides their

NARRATOR
This is a good move! As a tutor, it's better to ask questions than give answers. Also, notice how instead of talking about just the problem-at-hand, Alice has expanded the scope of the discussion to be about the concept of functions in general. Going general like this -- discussing the underlying concepts being used in a problem -- is more valuable to the student than just solving the problem.

BOB
It's a rule for assigning outputs to inputs.

ALICE
No dumbass.

BOB
What the hell Alice. I hate you.

NARRATOR
Alice was pretty nasty there, wasn't she? An effective tutor shouldn't insult their students. Let's try again.

Rewind time.

BOB
The question is asking me if this table of values represents a function or not.

ALICE
Ah, I see. OK! Can you tell me what a function is?

BOB
It's a rule for assigning outputs to inputs.

ALICE
Very close. Close enough for now. In this table, there are two columns. What do you think those represent?

NARRATOR

Notice that though the answer Bob provided was not quite on the nose, it was close enough that progress could be made.

BOB

I guess one is the input and the other is the output?

ALICE

Oh, that's very good! But which is which?

BOB

I don't know.

ALICE

OK. Well, by convention, we take the column labelled x to represent the input values while the column labeled y represents the corresponding output values.

Bob nods his head up and down indicating that he understands what's happening, but his face seems to indicate he's perturbed. But Alice suspects that Bob doesn't really understand and so follows with a question.

ALICE

Do you know what I mean when I say, "by convention?"

NARRATOR

Sometimes students will withhold their ignorance. But you can often tell when this happens through tells like body language or vocal giveaways. An effective tutor won't let this go.

BOB

My professor says that thing... "by contention," all the time but I have no idea what it means.

ALICE

No. Not "by contention." Rather "by convention." In this case, it just means that humanity has decided rather arbitrarily to make the letter x represent inputs to functions and the letter y represent outputs.

BOB

Well, wait, aren't x and y also the names of the lines in like, the plane?

ALICE

Yes, but let's not get into that now.

NARRATOR

Why not Alice? Bob is making connections. These sorts of connections are valuable for students and should be

ALICE
Let me ask again: what does the xy-plane have to do with functions?

BOB
Well, what about like graphs?

ALICE
Oh, good, I'm glad you mentioned graphs.

BOB
Right, because graphs are functions.

ALICE
NO! That's a category error. Graphs are curves and functions are rules for associating outputs to inputs. How can a curve be a rule?

BOB
Uh, what? I don't know.

ALICE
Then why did you say graphs are functions?

BOB
Uh I don't... uh... OK.

NARRATOR
But clearly it's not OK. Alice sure is being pedantic. I mean sure -- technically graphs are not functions. But they are excellent representations of functions. And the correspondence between functions and graphs is pretty tight! This sort of slip-up is small enough that Alice shouldn't get bogged down by it. How could Alice have done better?

Rewind time.

BOB
Right, because graphs are functions.

ALICE
Well almost. Graphs are great representations of functions -- kind of in the same way that a photograph of a tree is not the same as a tree. But we can totally work with graphs to study functions in the same way we can work with the photograph to study trees.

NARRATOR
A good analogy can go a long way.

Silence settles. Alice appears to be spacing out.

BOB
Alice, you there?

ALICE
Give me a second to think...

NARRATOR
Better to stop and think rather than say something stupid and rash.

ALICE
Is every curve that I draw in the plane the graph of a function?

BOB
Uh... I suspect that because you are asking that question, that the answer is "no" but I don't really know otherwise.

ALICE
Have you learned something called the "Vertical Line Test"?

BOB
Yes!

ALICE
OK! Great. Can you tell me what you know about it?

BOB
It's like, if uh, a straight line, like hits the graph more than once, then it's not a function.

NARRATOR
Alice is in a pedantic panic now. But she maintains her cool and chooses to charitably correct Bob's mistakes.

ALICE
OK... any straight line works for the *Vertical* Line Test?

BOB
Oh. Ha. No, of course the line

ALICE
OK, how about this. I'm going to draw two xy-planes. On one I want you to draw a curve which satisfies the Vertical Line Test, and on the other I want you to draw a curve which does not satisfy the Vertical Line Test.

BOB
OK

Alice draws two xy-planes, one on the left and one on the right. Bob proceeds to draw on left plane a squiggly curve which passes the Vertical Line Test, while on the right plane he draws an S shaped curve which doesn't pass the Vertical Line Test. Alice points to the left-plane:

ALICE
Now the Vertical Line Test says this one is the graph of a function and that one is not. Agreed?

BOB
Agreed.

ALICE
So why is this the graph of a function?

BOB
Because of the Vertical Line Test.

ALICE
What? No. That's circular thinking! You can't justify that the statement is true by using the truth of the statement. That would be like saying, "All squares are rectangles because all squares are rectangles," which is not good reasoning at all.

BOB
Uh, OK, sure.

ALICE
So why is this the graph of a function?

BOB
I don't know.

ALICE
OK. Let's take something even simpler because sometimes it's the simplest examples which are the most enlightening.

NARRATOR
It's true. Simple examples can isolate ideas and bring out misunderstandings of those ideas quickly.

Alice draws another coordinate plane and in the corner writes $f(x) = 2$.

ALICE
What does the graph of this function $f(x) = 2$ look like?

BOB
Oh, I know. It's a line.

ALICE
Can you be more descriptive?

BOB
It's a horizontal line.

ALICE
Yes! Can you draw the graph for me?

Bob draws the horizontal line $y = 2$.

BOB
Is that right?

ALICE
Yes, absolutely. Now suppose I erase the formula for the function...

Alice erases the $f(x) = 2$ bits.

ALICE
How could you use this graph to tell me the value of, let's say, $f(0)$?

BOB
Well, $f(0) = 2$ because it's always 2.

Alice takes a deep breath.

ALICE
Yes, but what is it about the graph that communicates that fact?

BOB

I don't know. I'm lost.

ALICE

Ugh. OK. You need to go back and review the relationship between a function and its graph. I don't have time to teach you things you should have learned in 11th grade. Come back to me when you're prepared.

Alice pulls her phone out and thumbs it violently, looking at the screen but not perceiving it. Bob is shocked, frustrated and sad.

NARRATOR

Woops. Frustration has boiled over and Alice has snapped. She is trying to place blame on Bob, or Bob's old teachers, or Bob's old schooling. But really, this is just a way of deflecting Alice's own responsibility as a tutor onto someone or something else. And look where we have ended up after this blowup: Alice is frustrated not just with Bob, but with herself, and Bob is totally dejected and left more confused than when he started. Let's try that again. Persevere Alice!

Rewind time.

BOB

I don't know. I'm lost.

ALICE

OK. In the equation $f(0) = 2$, what is the input and what is the output?

NARRATOR

Good job Alice. Back on track. Notice that in response to Bob's "I don't knows", Alice is digging deeper into the concept of a graph to try and find precisely where Bob's misunderstanding or missed connection is. This is good practice!

BOB

The input is 0 and the output is 2.

ALICE

Right. Now how does the graph communicate this?

BOB

I don't know.

ALICE

OK. You've seen your professor write things like "$f(x) = y$" before, yes?

BOB

Yeah. Oh, and sometimes my professor writes the y inside the f, but with like, a 1 at the top.

ALICE

Oh, like this: $f^{1}(y) = x$.

BOB

Yes! But what is that?

Alice realizes she doesn't know. She knows that f^{1} refers to the inverse of f, but she doesn't remember what it means. But she doesn't want to look ignorant so she confidently but wrongly replies:

ALICE

Oh, uh, that just means you take the reciprocal of f...

NARRATOR

Alice should not speak as though she knows what she is talking about if she does not. Rather, she should admit that she doesn't know -- there is no shame in that. Let's try this again:

Rewind Time.

BOB

Yeah. Oh, and sometimes my professor writes the y inside the f, but with like, a 1 at the top.

ALICE

Oh, like this: $f^{1}(y) = x$.

BOB

Yes! But what is that?

ALICE

Ummm, honestly, I'm not totally sure. I'll look it up after we finish this problem. But let's not get distracted! In $f(x) = y$, what is the input and what is the output?

BOB

The input is x and the output is y.

ALICE

Yes. So doesn't it make sense that the input is recorded on the x-axis and the output is recorded on the y-axis?

BOB

Hm, yes.

ALICE

OK. So let's look at $f(0) = 2$ again and compare it with $f(x) = y$. What's playing the role of x and y in this equation?

BOB

x is 0 and y is 2.

ALICE

Exactly. So where in this plane is the value of x equal to 0 and the value of y equal to 2?

BOB

Oh! It's right there...

Bob points at the point $(0; 2)$ in the plane.

BOB

...because that point is $(0; 2)$.

ALICE

Yes. And notice that point lies on the graph.

BOB

OK.

ALICE

So the fact that the point lies on the graph is what is communicating that $f(0) = 2$.

BOB

Oooh.

ALICE

So now let me see if you really understand. With this graph, can you tell me what $f(\ 1)$ is?

BOB

Still 2.

ALICE

OK, but what is it about the graph that is communicating that fact?

BOB

It's this point.

Bob points at $(\ 1; 2)$.

ALICE

Yes! And what are the coordinates of that point?

BOB

$(\ 1; 2)$.

ALICE

Now please observe, both the points $(0; 2)$ and $(\ 1; 2)$ lie on the graph, right?

BOB

Sure.

ALICE

But another way we can write these points is $(0; f(0))$ and $(\ 1; f(\ 1))$, right?

BOB

Sure. But who cares. f of anything is always 2.

ALICE

That's true, and do you see how on this graph, this horizontal line, it doesn't matter what value of x you choose. The point on the graph with that value of x as its x-coordinate has a y-value of 2.

BOB

Wait what?

NARRATOR

Alice realizes that her last sentence was pretty complicated and that she said standing in place with a rather flat affect. She realizes that she could probably demonstrate the idea better with some relevant body movements and markings on the drawing.

ALICE

No matter what value of x

graph in a point. She marks this intersection point by going over it with the marker a bunch.

BOB
So it's the y-coordinate of that intersection.

ALICE
OK, great. Now let's look at this other graph. This fails the Vertical Line Test, right?

BOB
Yes.

ALICE
Can you draw me a vertical line which causes the Vertical Line Test to fail?

BOB
Yep.

Bob draws a vertical line which intersects the S-shaped curve in three places.

ALICE
Great. So the Vertical Line Test says that this cannot be the graph of a function, right?

BOB
Yep.

ALICE
But then, why does this vertical line, which intersects the curve in three places, mean that this is not the graph of a function?

BOB
Well, there's not just one y-value. There's three.

ALICE
Aha! That's right.

BOB
So what?

ALICE
So remember what you said a function was at the very start?

BOB
It's a rule for assigning outputs to inputs.

ALICE
Yes. And I said that was close to the right definition. But we see that the Vertical Line Test imposes one more condition.

BOB
Oh yeah, there can only be one output.

ALICE
YES! So what then is the definition of a function?

BOB
It's a rule which assigns a unique output to an input.

ALICE
On the nose.

BOB
That's it? I've been saying that the whole time!

ALICE
No. You really haven't.

NARRATOR
It's good to be firm with the ungrateful. Let's face it: Bob should be kissing Alice's feet.

BOB
You could have just told me the right definition from the start.

ALICE
Sure, but you learned more this way and I get paid by the hour.

BOB
Whatever. OK, so then the answer to the homework problem is easy.

Bob points back at the table:

x	y
1	3
2	3
3	4
3	-4
4	3

The column marked x_{BOB}

Lessons to Learn from "An Easily Imagined Encounter..."

There are a number of moments in *An Easily Imagined Encounter Between a Tutor and Student* which are examples of good and bad pedagogical practice. Some of the principles were noted by the narrator, and some were not. Here is a non-exhaustive list of possible pedagogical principles we might take to represent good practice.

1. Be attentive.
2. Guide students to solutions.
3. Be polite.
4. Ask questions. Don't just give answers.
5. Go general to ascertain where a student's misunderstanding of an idea lies.
6. Work with and build from existing understanding.
7. Watch for cues from body language.
8. Encourage students to make connections.
9. Ruthlessly squash fundamental misunderstandings.
10. But don't be overly pedantic. Find the right balance between being correct and being a jerk.
11. Analogies to familiar objects or ideas can be illuminating.
12. It's ok to stop and think.
13. Get students actively involved.
14. If a student isn't understanding an idea, try illustrating it with the simplest possible example.
15. Don't let frustration (which will inevitably come) get the best of you.
16. If you don't know something well enough, be open and honest in admitting it.
17. Body movements enhance the exposition of an idea.
18. Be patient.

☐ **Exercise 1:** Rank yourself on your proficiency in the practice of each of these principles based on your self-perception and, perhaps, your past experience with tutoring or teaching in general. Fill in the following table with a rating of yourself on a scale from 1 to 5, 5 representing mastery and 1 representing incompetence.

#	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
proficiency																		

Perhaps if you see yourself as strong in the use of one of these principles, you can use it more often. And perhaps if you see yourself as weak in one, you can try to be conscientious in how you might improve.

☐ **Exercise 2:** Are there other principles of good teaching practice you believe were not listed above, or even present in *An Easily Imagined Encounter...*?

Author: Taiyo Inoue

2.2 Different Approaches

[A.K.A. Two roads diverged in a yellow wood ....] 1

Product Rule Discussion

Tutor: Let's rst look at $2^3 2^2$. How can we simplify this?

Student: I'm not sure tbh.

Tutor: Okay, well what does 2 cubed really mean?

Student: It means we have three 2's.

Tutor: Okay, so let's write $2\ 2\ 2$. And what does 2 squared mean?

Student: Two times two.

Tutor: Okay, so we have those rst three 2s, multiplied by two more, so $2\ 2\ 2\ 2\ 2$. How many 2s total?

Student: Five.

Tutor: Okay, so how do we write our answer?

Student: 2^5.

Tutor: Great! So now looking back at the original problem, how can we get 5 from the exponents 2 and 3?

Student: You can add them!

Tutor: Exactly! In fact, this always works - anytime you see the same base (2 in our example) multiplied, you can add the exponents. This is a good thing to have jotted down in your notes, along with a few examples like these.

Tutor: Now let's look at another exponent example. (Go to Power Rule Discussion)

Power Rule Discussion

Tutor: Okay, now let's consider $(2^1)^3$. How would you solve this?

Student: Well, like I said I think it would be 2^4.

Tutor: Okay, what's another way to solve it, maybe going step by step?

Student: Hmm, well $2^3 = 8$, so then we have $(8)^1$ which is just 8.

Tutor: Great, you followed order of operations there! Now what does 2^4 equal?

Student: Uh, well, my calculator here says 16.

Tutor: Okay, so we have two di erent answers here. Which method of solving do we believe?

Student: Well, I guess the one where we get the answer 8.

Tutor: I agree with you; it's easier to believe something when we can work out the steps and see each move. So I think we just need to observe what's happening with the exponents here so we can

Tutor: Okay, and the same for the second set of parentheses. So, now we move outside of the parentheses. What can you do there?

Student: Well, we just learned that rule that exponents multiply but there's a lot more here.

Tutor: Sure, so let me show you a trick. We can rewrite the rst set of parentheses as $(x^3y^1z^7)^1 = (x^3)^1(y^1)^1(z^7)^1$.

Student: Okay, I can work with that. We get $x^3y^1z^7$.

Tutor: Great! Can you do the same with the second set of parentheses?

Example Tutoring Problem: Exponential vs. Polynomial Functions

One problem you might encounter with students is a di culty grasping the di erence between exponential functions and polynomials.
For instance, consider the functions:

$$f(x) = x^2$$
$$g(x) = 2^x$$

In MTH 100, a student might be asked to draw $g(x)$ and draw a parabola like that of $f(x)$. Another student in MTH 210 (Calculus 1), might be asked to di erentiate $g(x)$ and use the power rule.

In both cases, students are seeing a pattern in that we have a base and an exponent but they are not understanding that it matters where our variable is placed within the expression. Please answer the following questions. You may choose to consider how you would answer for a MTH 100 or MTH 210 student or both.

1. What are your rst thoughts about how you might help a student understand that fundamental di erence?

2. What are some leading questions you could ask them?

[Continued on next page!]

3. What are some tasks you can ask them to do to guide them towards understanding?

4. How would you try to see whether they have internalized this understanding, before moving on?

5. Often a student who asks, Why are we learning this at all? Why does it matter? How would you respond to this?

CHAPTER 3

Concept Inventory for Math Courses

The last part of this packet is simply an inventory of concepts that cover each course we teach and some additional resources you might nd useful if you are ever stumped or need to brush up on some concepts. If you are tutoring students enrolled in a particular course, we highly suggest that you::

1. Read through the concept inventory for that course and verify that you are familiar with these concepts or not. If not, it is advisable to brie y brush up on this material.
2. Use this inventory as a guide to help students in their study habits and what they need to know for their course and exams. These inventories can be thought of as a sort of Table of Contents or Study Guide for each course. Disclaimer: these inventories contain most of the course concepts, but content and emphasis does vary slightly depending on instructor.
3. Request the syllabus from the instructor and ask for any additional resources.

Remember: as a tutor, you aren't expected to remember everything in each course your students are in. You are expected toguide them towards understanding. So it's okay to say you don't know or you don't remember, but you should always give students resources or, if possible, let them know that you are going to look into it and can follow up with them later if that's useful. You are modeling the behavior of a good student: you don't give up and you seek information or clari cation elsewhere if needed.

3.1 MTH 100

1. Functions
   a) Do you know what a function is?
   b) Given a relation or a table of inputs and outputs, can you identify which represent functions and which do not?
   c) Can you tell when a curve in the plane is the graph of a function?
   d) What is the domain of a function? Are you comfortable nding domains?
   e) What is the range of a function? Are you comfortable nding the range, particularly if you have the graph of the function in front of you?

   f) Do you know how to add, subtract, multiply and divide functions?
   g) Do you know how to do the composition of functions?
   h) What's a piecewise function? How to work with them? How to graph them?
   i) Do you know what it means for a function to be invertible?
   j) Do you know how to tell if a function is invertible from its graph?
   k) Do you know how to compute the inverse of an invertible function?
   l) Be able to describe the relationships of inverse functions.
   m) What are some examples of functions which are invertible and which are not?
   n) Do you know how to compute the average rate of change of a function over some interval? Do you know how to interpret this quantity?

2. Graphs of Functions

   a) Do you understand how to interpret the graph of a function?
   b) If we apply some small change to $f(x)$, like adding 4 to $f(x)$ or subtracting 3 from x as in $f(x - 3)$, do you understand how the graph changes?
   c) What about if we multiply $f(x)$ by some number like 3? What happens to the graph?
   d) Know how to look at a graph and give the equation of the function you see.
   e) Know how to identify y-intercepts and x-intercepts of any mathematical equation.
   f) Know how to find the slope of a line.
   g) Given a line, know how to find lines parallel and perpendicular.
   h) Understand the relationship between shifting graphs of functions and their mathematical notation (transformation of functions).
   i) Know how to identify the vertex and axis of symmetry of quadratic equations.
   j) Know how to graph piecewise functions, paying careful attention to the domain restrictions.
   k) Know how to determine whether a function is increasing or decreasing, and how to determine end behavior.

3. Linear Expressions

   a) Do you know how to find the equation of lines given the slope and a point on the line?
   b) Do you know how to find the equation of a line given two points on the line?
   c) Do you know how to model phenomena using linear expressions?
   d) Do you know how to solve inequalities that involve linear expressions such as $2x + 3 > 5$.

4. Quadratic Expressions

   a) Do you know what a quadratic function is?
   b) Do you know what the graph of a quadratic function looks like?
   c) What do we mean by the roots of a quadratic?
   d) When we have an equation involving a quadratic, like $x^2 + 6x + 9 = 1$, why do we strongly prefer to 0 on the right side instead of some other number?

e) Do you know how to factor a quadratic?

f) Do you know how to complete the square?

g) Do you know the quadratic formula?

h) What's an irreducible quadratic? How can we detect them?

i) How can you use a gadget called the discriminant to tell if a quadratic has 0, 1 or 2 real roots?

j) Know how to find the solutions/zeros of quadratic equations.

5. Equations involving radicals

   a) An equation involving radicals is something like $\sqrt{2x+3} = x$.

   b) What are "fake solutions?" Why do they arise when working with this sort of equation?

   c) Can you solve $\sqrt{2x-3} = x$?

6. Polynomial expressions

   a) Can you identify when an expression is a polynomial and when it isn't?

   b) What do we mean by the "root" or "zero" of a polynomial?

   c) What do we mean by a repeated root?

   d) What is the degree of a polynomial?

   e) Does a polynomial of degree n always have n real roots when taking into account repeated roots?

   f) Does a polynomial of degree n always have n complex roots when taking into account repeated roots?

   g) What is a good approximation for a polynomial $p(x)$ when $|x| \gg 0$.

7. Rational expressions

   a) What's a rational expression?

   b) What's a vertical asymptote? How do we find them? Does every graph of a rational expression have a vertical asymptote?

   c) What's a horizontal asymptote? How do we find them? Does every graph of a rational expression have a horizontal asymptote?

   d) After we find asymptotes, do you feel comfortable sketching the graph of a rational expression?

8. Exponential functions

   a) Can you recognize when a function is exponential?

   b) Be able to explain the components of an exponential equation.

   c) What is meant by exponential growth? What is meant by exponential decay?

   d) What are some phenomena measured by exponential growth or exponential decay?

   e) Do you know how compound interest works?

9. Logarithms

a) How are logarithms related to exponentials?

b) Know how to use logarithms to solve equations involving exponential expressions.

c) Be comfortable using the properties of logarithms in solving.

d) What's the cool thing we can say about an expression like $\log_b(a^c)$?

e) What about $\log_b(ac)$?

f) What about $\log_b(\frac{a}{c})$?

10. Trigonometry

a) Know how to describe periodic functions and their characteristics.

b) Know how to use the periodic circle to solve for various angle and side lengths of a triangle.

c) Be comfortable with the three main trigonometric functions - sine, cosine, and tangent.

d) Know how to use trigonometric properties to work with non-right triangles.

e) We have two ways of measuring angles degrees and radians. Do you understand this?

f) Do you know how to convert an angle in degree to radians? Vice versa?

g) Do you know the de nition of $\sin( )$ and $\cos( )$ as it relates to the unit circle?

h) Do you know the values of $\sin( )$ and $\cos( )$ at particularly special angles like $0 ; 30 ; 45 ; 60 ; 90$?

i) Do you know how to use these values to nd the values at other special angles in other parts of the unit circle? For example, do you know what $\sin(210)$ is? How about $\cos(120)$?

j) How do we de ne $\tan( )$? What about $\sec( ); \cot( ); \csc( )$?

k) How do these trig functions relate to right triangles?

l)

3.2 MTH 107:

1. Types of Data
   a) Do you know what the terms continuous vs. discrete mean and how they apply to data?
   b) Do you know what the terms quantitative vs. qualitative mean and how they apply to data?
   c) What is the di erence between a samples and a population?
2. Measures of Central Tendency
   a) What is a mode and why is it useful?
   b) What is a median and why is it useful?
   c) What is a mean and why is it useful?
   d) What does it mean to be in the middle? How do we measure this?
   e) Is there a better or worse way to measure the middle of a data set?
3. Measures of Spread
   a) What is the range?
   b) What does standard deviation measure?
   c) What is the interquartile range (IQR) and what does it represent?
   d) How do we measure the spread of data? Is there one method that works better than another, or describes data better?
4. How we describe data
   a) Graphical representations: do you know how to represent di erent data types data graphically? Can you describe how graphical representations can be misleading?
   b) What is a distribution? What is the realtionship between distributions and their relations to graphical representations of data, measures of central tendency, and measures of spread?
   c) What does it mean for a distribution to be symmetric vs. right (positively) and left (negatively) skewed? Can we see this with measures of central tendency?
5. Measures of Error
   a) Does a sample look exactly like the population? How do we understand this?
   b) How good is the sample mean at estimating the population mean? Is the sample mean exactly equal to the population mean?
   c) How do we measure the typical error of the sample mean in estimating the population mean?
   d) What is standard error and why do we need it?
   e) Why does standard error have a $\sqrt{n}$ in the denominator? What happens to the standard error as the sample size gets large? What does this mean about the error of the sample mean in estimating the population mean?

   f) What is a con dence interval?
   g) What does a z-values or t-values represent? When do we use z versus t?

6. Hypothesis testing
   a) How do we see whether our data conforms to our expectations?
   b) How do we quantify this statistically? Can we say that our statistics prove something?
   c) What are the ve steps of hypothesis testing
   d) What is a null hypothesis vs. alternative hypothesis?
   e) What is a signi cance level?
   f) What is a critical (or threshold) value?
   g) What does one-sided (one-tailed) vs. two-sided (two-tailed) mean?
   h) What does a z-values or t-values represent? When do we use z versus t?
   i) What does it mean for a result to be statistically signi cant ?

7. Relationships between data: correlation and regression
   a) What is correlation?
   b) What values does a correlation coe cient take?
   c) How is correlation calculated?
   d) What is causation?
   e) Give an example where correlation does not imply causation.
   f) What kind of data can we use for linear regression?
   g) Conceptually, what is a regression line? (What does it represent?)
   h) What two conditions must a line satisfy to be considered a regression line?

8. Probability
   a) What does probability mean?
   b) How do we measure probability?
   c) What is the range of a probability?

3.3 MTH 210:

1. Limits
   a) What do the concepts of limits and continuity mean?
   b) Can you compute limits of a variety of functions?
   c) What makes a limit hard to compute?
   d) What is an indeterminate form? What do we do with indeterminate forms?
   e) Can you determine regions of continuity?
   f) Can you determine end behavior using limits?

   g) Can you use l'Hopital's rule to compute a limit?

2. Derivatives
   a) Can you use derivative rules to compute the derivative of any function?
   b) Can you use a composition of derivative rules to compute more complicated derivatives?
   c) Do you know what the de nition of a derivative is?
   d) Can you use the de nition of derivative to nd a derivative of a function?
   e) Can you determine regions of di erentiability?
3. Behavior of functions and graphs
   a) Can you use the 1st derivative to sketch graphs of functions?
   b) Can you use the 2nd derivative to sketch graphs of functions?
   c) Can you determine where a function is increasing/decreasing?
   d) Can you determine where a function is concave up/dow?
   e) Can you use the 1st and 2nd derivative tests to classify extrema?
   f) Can you de ne critical point, in ection point, local/global extrema?
4. Applications
   a) Can you explain the relevance of derivatives in dynamic systems in the real world?
   b) Can you translate problems involving dynamic systems into problems of calculus?
   c) Can you use the derivative to solve applied problems involving dynamic systems?

3.4 MTH 211:

1. Integration
   a) Can you de ne a de nite versus inde nite integral?
   b) Can you nd antiderivatives using a variety of rules?
   c) Can you nd areas of regions under curves and between curves?
   d) Can you approximate the area under a curve using Riemann sums?
   e) Can you nd the exact area under the curve using the de nition of an integral?
2. Sequences and Series
   a) Can you evaluate in nite sequences and series and be able to determine whether they converge or diverge?
   b) Can you explain what a sequence and series mean?
   c) Can you write a sequence or series using mathematical notation?
3. Applications:
   a) Can you apply de nite integrals in the solution of practical problems in geometry, science and engineering?
   b) Do you understand di erential equations and how we use them in mathematical modeling?

3.5 MTH 212:

1. You should know about vectors.
   a) What kinds of physical quantities do vectors represent?
   b) How do we notate vectors in this course?
   c) What is a unit vector? How do we nd a unit vector pointing in a direction?
   d) What is the distinction between points and vectors?
   e) Do you know how to compute the dot product of two vectors? The cross product of two vectors? Does it make sense to take the cross product of vectors in R^2? In R^3? In R^n?
   f) How does the dot product give rise to notions of angles and lengths?
   g) What does the dot product output? How to interpret its output?
   h) What does the cross product output? How to interpret its output?
2. You should know about lines and planes.
   a) Do you know how to nd a line in the plane or space with a given direction and a given point which the line passes through?
   b) Do you know how to nd a line given two points which lie on the line?
   c)

b) What are some physical quantities they can represent?

c) What are partial derivatives? What do they represent? How about directional derivatives more generally?

d) What are level sets of a scalar field? What are the level sets of a the scalar field $f(x;y;z) = x + y + z$? What about $f(x;y;z) = x^2 + y^2$? What about $f(x;y;z) = x^2 + y^2 + z^2$?

e) What is the gradient of a scalar field? Why do we care about it? How do we interpret it?

f) How is the gradient of a scalar field related to the level sets?

g) How can we use the gradient to or tangent planes to surfaces?

h) Is every vector field the gradient of some scalar field?

i) How can we maximize or minimize scalar fields? On a closed and bounded region, how do you find the maximum and minimum values of a scalar field?

5.22

x;y9091 Tf 4.732 -Tf 95123.273 628.216188.643hat are3e5dd333(onde73(they)63auan)5ctor e6d theserives?g

l) Why is the curl of a vector eld important for line integrals?

m) What is Green's Theorem? Why is it awesome?

7. Do you like surface integrals?

   a) Do you know how to parameterize a surface? What if the surface is the graph of a scalar eld, like $z = f(x;y)$? What if the surface is a constant coordinate surface in some coordinate system? What if the surface arises as a surface of revolution around some axis?
   b) Why do we care to parameterize surfaces?
   c) How do we calculate surface area of parameterized surfaces?
   d) How do we calculuate integrals of vector elds over parameterized surfaces? How do we interpret such things?
   e) What does an orientation on a surface mean?
   f) How can we nd a normal vector to an oriented surface in R^3 which agrees with the orientation?
   g) What is a closed surface? What are examples of closed and non-closed surfaces?
   h) What is an orientable surface? Is every surface orientable? No? What's an example of a non-orientable surface?
   i) What is the divergence of a vector eld? Why do we care?
   j) State the divergence theorem.
   k) What is Stokes' theorem? Why do we care?

3.6 MTH 215:

1. First-order Ordinary Di erential Equations:

   a) What is the most general form for a rst order di erential equation?
   b) Do you know how to solve a di erential equation of the form $y^0 = f(x)$?
   c) Do you know how to solve an initial value problem of the form $y^0 = f(x)$, $y(a) = b$?
   d) Do you know how to present an antiderivative of $f(x)$ as an integral? Do you know why we might want to do such a thing? What is the solution to $y^0 = \exp(\ \ x^2)$, $y(0) = a$?
   e) What is a separable equation? How do you solve them?
   f) What is a slope eld? How do you nd the slope eld for a rst order DE? How do solutions to the DE relate to the slope eld?
   g) Do you know what a general rst-order linear di erential equation is? Do you know why we use the word linear to describe them? Do you know how to solve them using the method of integrating factors?
   h) Do you know how to apply rst-order equations to problems like free-fall with air resistance, or mixing problems, or cooling problems?

2. Second-order Linear Ordinary Di erential Equations

d) Do you understand that matrix multiplication is not commutative? Do you have an example of a pair of square matrices A and B where $AB \neq BA$?

e) Do you know what the identity matrix is? What is its important property?

f) Do you know what the inverse of a matrix is? Does a matrix always have an inverse? If not, what is a way to detect when a matrix has an inverse?

g) Do you know how to compute the inverse of a matrix?

h) Do you know how to solve the equation $Ax = b$ where x and b are column vectors?

i) Under what conditions is there exactly one solution to $Ax = b$?

j) Do you know what the elementary row operations are?

k) Do you know what the reduced-row echelon form of a matrix means?

l) Do you know what the determinant of a square matrix is? Do you know how to compute it (cofactor expansion)?

m) Do you know what eigenvectors and eigenvalues of a matrix are? Do you know how to find them?

n) Do you know how to work with complex eigenvalues?

o) Do you understand the issues that may arise with repeated eigenvalues?

p) Do you know what is meant by the algebraic and geometric multiplicity of an eigenvalue?

q) Do you know what a generalized eigenvector is?

r) Do you know what the exponential matrix is? Do you know how to compute it if you have an $n \times n$ square matrix with n linearly independent eigenvectors?

5. Systems of First Order Linear Differential Equations

a) Do you know how to express the general form of these systems?

b) What does it mean for such a system to have constant coefficients?

c) What does it mean for such a system to be homogeneous?

d) Do you understand the superposition principle for homogenous linear systems?

e) Do you understand the notion of linear independence of solutions?

f) Do you understand how to solve $x' = Ax$ where c)

3.7 Other Course Resources

These lists can easily become out-of-date quickly, but new tools and resources are being developed all the time. A few notes:

Ë

Appendices

APPENDIX A

Solutions or Discussion Ideas for the Homework Assignment

(for the worksheet Example Tutoring Problem: Exponential vs. Polynomial Functions):

1. Leading questions/tasks you can ask them (slightly change depending on the level of student):

   - Choose several values of x and plug it into $g(x)$. Make sure to ask them to us both positive and negative x values, if they do not do this immediately. Do your values match your graph or derivate expression? Why? What should the graph or derivative's functional form be, roughly?
   -

Acknowledgements

The authors wish to acknowledge support from the CSU Chancellor's Office Graduation Initiative 2025 funding which supported the development of this material. CODED MEMO B 2019-03 specifically designated funds for use for programs focusing on student engagement, support for students and faculty in courses redesigned to meet the requirements of EO 1110, investing in data-driven decision making and technology to advance successful academic support programs, and offering professional development programs that identify additional opportunities for closing equity gaps. Some of this funding was requested to develop a Mathematics Tutor Training Workshop, led by mathematics faculty, that would supplement existing tutor training in our tutoring center on campus.